1. Consider the function defined by
2. Show that *T* is a linear transformation

To be linear, T has to hold under addition of two vectors and scalar multiplication. For addition of two vectors and ,

Then, for multiplication with a scalar

Thus, *T* is a linear transformation.

1. Write down the standard matrix for the transformation *T* (denoted by [*T*]).

The standard matrix can be found by transforming the unit vectors in by *T*:

Thus, the standard matrix for *T* is

1. Is ? If it is, find the vector that is mapped to by *T*.

To see if the vector is in the range of *T*, the following system of linear equations has to be consistent:

the solution to which is the vector Since the system of linear equations has one solution, is in the range of *T*. The vector is also the vector mapped to by *T*, as required.

1. Consider the following sets of vectors,

both of which form an ordered basis for

1. Find the coordinates of

with respect to the ordered basis B.

To do this, the linear system of equations based on basis B

where scalars are the multiples of the vectors in B. The solution to this system is the vector

1. Find the change of basis matrix

Expressing each vector in B as a linear combination of vectors in C:

From the scalars in these equations, the change of basis matrix can be found to be

1. Using parts (i) and (ii), find the coordinates of ***v*** with respect to the ordered basis C.

Matrix multiplication can be used to find

1. Let C and D be matrices and . Suppose that and that det Show that, if is an eigenvector of C with corresponding eigenvalue , then ***x*** is an eigenvector of C, and find the corresponding eigenvalue.

Since det, D is an invertible matrix. Furthermore, the assumption can be written as

Because ,

Since D is invertible, it can be multiplied on the left side to make

Then, since is a scalar, it can be rearranged to make

Since ,

Thus, ***x*** is an eigenvector of C with corresponding eigenvalue , as required.